Indian Statistical Institute, Bangalore Centre
B.Math. (I Year) : 2011-2012
Semester I : Backpaper Examination
Probability Theory I

December 2011 Time: 3 hours. Maximum Marks : 100

Note: State clearly the results you are using in your answers.

1. (10+10 = 20 marks) Box A contains 2000 items of which 5 percent are defective, and Box B contains 500 items of which 40 are defective. One of the boxes is chosen at random, and from it a single item is selected at random.

   (i) What is the probability that the selected item is defective?

   (ii) If the selected item is found to be defective, what is the probability that it came from Box B?

2. (10+5 = 15 marks) Let X, Y be independent discrete random variables each having a geometric distribution with parameter $0 < p < 1$. Let $Z = \min\{X, Y\}$.

   (i) Find the discrete density function of the two-dimensional discrete random variable (X, Z).

   (ii) Are X and Z independent?

3. (15 marks) Let X, Y be independent discrete random variables such that $P(X = j) = P(Y = j) = \frac{1}{N}$ for $j = 1, 2, \ldots, N$ where N is an integer. Compute the discrete density function of $X + Y$.

4. (5+5+5 = 15 marks) Let X be a discrete random variable having Poisson distribution with parameter $\lambda > 0$.

   (i) Find $\mathrm{E}(X)$.

   (ii) Find the probability generating function of X.

   (iii) Find the moment generating function of X.

5. (13+7 = 20 marks) Let $h : [0, \infty) \to [0, \infty)$ be a continuous function such that $\int_0^\infty h(y)dy = \infty$. Define

$$\begin{aligned} F(x) &= 1 - \exp(-\int_0^x h(u)du), \quad \text{if} \quad x \geq 0, \\ &= 0, \quad \text{otherwise.} \end{aligned}$$

(i) Show that F is a distribution function.

(ii) Find the corresponding probability density function.

6. (15 marks) Let X have $N(\mu, \sigma^2)$ distribution where $\mu \in \mathbb{R}, \sigma^2 > 0$. Let $Y = aX + b$ where $a \neq 0, b$ are real numbers. Find the probability density function of Y.